Evidential Multiple Choice Questions

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Abstract. One of the most common and computably tractable ways of evaluating the knowledge of a student is through the use of questionnaires with multiple choice questions (MCQ), where students must express a precise choice to answer a question, without leaving room for ambiguities or doubts. The problem is that sometimes the student doesn’t really know the answer or cannot decide between the possible choices, even if he is able to discard some of them. We propose an alternative MCQ that, using belief function theory, allows the student to state his answer in an imprecise way, indicating to which degree each possible choice represents the correct answer. This way we get to model the ignorance and uncertainties of the learners, allowing an Intelligent Tutoring System (ITS) to gather a richer student model.

1 Introduction

Thanks to their intuitive interaction and computability, MCQs are arguably the most common method used by ITSs to measure student knowledge acquisition. They are seldom used by educational adaptive hypermedia systems [1] to update the knowledge model of the students and adapt the content and presentation of a learning object to the specific state of a user (student) model. Overlay Models are used to represent the mastering and misconceptions of the domain concepts studied [6, 7].

Even if MCQs have been widely studied in psychometrics and several theories have been proposed (Classical Test Theory, IRT [5]) in order to optimize trait estimation and their use in CAT (Computer Aided Testing), their interaction prevent students from stating their hesitations.

When passing a MCQ test, a student has to choose among the possible answers presented in a question. The choice has to be precise even if, as sometimes occurs, the student is not entirely convinced by his own answer. It is normal for a student to find himself in a situation where a question appears to be ambiguous according to the options presented. He may be able to recognize some of the options as incorrect, but not be able to establish the correctness of all of them. He can also be facing a question to which he is certain he does not know the answer, or to which, in his opinion, the possible choices presented do not seem to answer the question. When in doubt, a student must make a blind choice among the answers that, in his view, are not completely wrong.

This situations cannot be treated accordingly by classical MCQs, they lack of a way for students to express doubt, ignorance, and uncertainty. By constraining the students to precise answers, noise is gathered and considered as an input to update the student model. Invaluable information as to the real state of knowledge of the student is then lost. The concepts that have not been fully acquired by the students, the concepts that the students themselves are certain they possess and the concepts that the students have wrongfully considered as acquired could have been identified.

Some authors have proposed the extension of classical MCQs by allowing the student to specify a degree of global confidence on their answers (by choosing a label) or select alternative possible choices (by suggesting an alternative choice that could be the correct answer to the question) [2]. These approaches provide some flexibility, but they are still too restrictive to represent imperfect degrees of knowledge.

We propose evidential multiple choice questions (ev-MCQs), an alternative to classical MCQs, that use belief function theory to allow students to state their answer denoting their imprecise knowledge of the evaluated subjects, indicating to which degree each possible choice can represent the correct answer. ev-MCQs can then be used to diagnose knowledge acquisition and misconception. This way, students are directly involved in the evaluation and further modeling of
their own knowledge, and ITSs can then be able to gather a richer learner model closer to the actual state of the student knowledge.

2 Belief Function Theory

Belief Function Theory (or Dempster-Shafer’s evidence theory) [3] is a mathematical theory that generalizes probability theory by abandoning the additivity constraint. Instead of assigning a probability mass to atomic elements only, belief masses can also be associated to subsets of elements.

The frame of discernment \( \Theta \) is the set of all possible elements. A mapping \( m : 2^\Theta \rightarrow [0, 1] \) assigns a belief mass to the subsets of \( \Theta \) and is called a basic belief assignment (BBA). The value \( m(A) \) represents the fraction of the belief mass unit assigned to the subset \( A \subseteq \Theta \), with the constraint \( \Sigma_{A \subseteq \Theta} m(A) = 1 \).

The subsets \( A \) of \( \Theta \) such that \( m(A) > 0 \) are called focal elements. The case where all focal elements are singletons is the bayesian belief assignment, the particular case of probabilities.

Perfect knowledge of the state \( A \subseteq \Theta \) of the world is represented by assigning the unit of belief mass to \( A \). On the other hand, total ignorance is represented by the vacuous belief function, an assignment of the total mass unit to the frame \( \Theta \).

In Dempster-Shafer’s interpretation, the frame of discernment is considered to be complete (closed world assumption) and \( m(\emptyset) = 0 \). In Smets’ interpretation, called the Transferable Belief Model (TBM), belief masses are considered as numerical representations of the state of belief of a rational agent [4] and \( m(\emptyset) > 0 \) can model the fact that the current state of the world is not among the elements of the frame \( \Theta \) (open world assumption). We follow Smets’ interpretation of belief functions.

3 Evidential Multiple Choice Questions (Ev-MCQs)

3.1 Specification of a MCQ question

A multiple choice question can be described by a triplet \( < \Theta, R, r_{ok} > \), where \( \Theta \) is a set of propositions that can be considered as answers to the question, \( R \) is a set of subsets of \( \Theta \) representing the possible choices that the student can make to answer the question such that \( R \subseteq 2^\Theta \), and \( r_{ok} \) is the correct answer for the question, \( r_{ok} \in R \). We will note as \( r_i \) the possible choices presented in \( R \).

![Fig. 1. Examples of MCQ.](image)

(b) An ev-MCQ-SR.
We recognize 2 types of MCQ questions. MCQ-SR (Single Response) consider only singletons of $\Theta$ as members of $R$ ($R = \Theta$); True or false questions are part of this category. MCQ-MR (Multiple Response) have $R$ made up of choices composed of subsets of elements of $\Theta$ (see Fig. 1(a)); $r_{\text{ok}}$ is then a subset of $\Theta$.

We can see MCQ-MR questions as a collection of MCQ-SR, where each proposition in $\Theta$ can be considered separately as a true or false question (as we will explain in the next subsection).

3.2 Belief assignment on an ev-MCQ question

A student answers a question by distributing a belief mass unit among the options presented ($R$); he is not forced to assign all of his mass to a single option $r_i$, he can distribute it as he wants. Also, he can leave some mass unassigned indicating his ignorance or lack of confidence on his answer. The acquired freedom allows the student to involve some of the choices that otherwise he wouldn’t consider, enriching his answer.

We will first explain the belief assignments for the particular case of a MCQ-SR question, where there is only one correct answer $r_{\text{ok}} \in \Theta$. $R = \Theta$ is the frame of discernment of MCQ-SR questions (see Fig. 2(a)). We can represent an answer as a BBA among the elements $\theta_j \in \Theta$. The unassigned mass can then be associated to $\Theta$ itself. We will only have singletons and the frame as possible focal elements of the BBA that represents the answer.

![Fig. 2. Frames of Discernment.](image)

The approach for the belief assignment of MCQ-SR questions cannot be followed on MCQ-MR questions, since multiple propositions in $\Theta$ can be part of the correct answer (e.g. $r_{\text{ok}}$ on the MCQ-MR shown on Fig. 1(a) is composed of $a$, $b$ and $d$). For this type of questions, we consider each proposition $\theta_j \in \Theta$ separately, taking a MCQ-MR as several virtual true or false MCQ-SR questions having an independent frame of discernment $\theta_j$ with only two elements stating that the $j$-th proposition is either true or false. The example shown on Fig. 1(a) will induce 5 independent frames of discernment as shown on Fig. 2(b).

In MCQ-MR questions, the student will assign a mass $m(r_i)$ (possibly 0) to every $r_i \in R$. These options contain some elements $\theta_j \in \Theta$. The presence of a proposition $\theta_j \in r_i$ will be interpreted as the statement that that proposition is a correct answer for its question, the mass assigned to $r_i$ by the user will be associated to the true singletons in the frames of each $\theta_j \in r_i$ and to the false singletons in the frames of each $\theta_j \not\in r_i$.

If a student doesn’t think the answer to the question is represented by any $r_i$, he won’t assign any mass. If he is sure the answer is among some of the choices, he will distribute all his belief among them, not leaving any mass unassigned. If he is not totally confident of his answer, he can leave some portion of mass unassigned, representing in fact his ignorance of the actual answer (we will note this ignorance mass $u = m(\Theta)$). The value of $c = 1 - u = \Sigma_{\theta_j \in R} m(r_i)$ represents the confidence of the student that the answer resides among the chosen options.

Some MCQ-MR questions have as possible choices the statements “all of the above” and “none of the above”. The assignment of belief mass to these options will be interpreted as the assignment to the true singletons of all $\theta_j \in \Theta$, and to the false singletons of all the $\theta_j \in \Theta$ respectively.
3.3 Example

ITS have always used traditional *widgets* (as radio buttons and checkboxes) to present the MCQ questions. These controls are too restrictive, and they force the students to give a precise answer. We needed to implement a novel way to control the amount of mass that could be assigned to every choice $r_i$.

An example of an answer of an evidential MCQ question can be seen in Fig. 1(b). It is the case of a MCQ-SR question, where $R = \{r_1, r_2, r_3, r_4, r_5\}$, and $\Theta = R$.

On the right side of the lemma of the question we can see a bar that shows the mass that has been assigned by the student ($c$) and the mass that has not been assigned ($u$), this bar only shows these mass values and cannot be modified by the student. These values are updated automatically when the student changes his answer by interacting with the bars that control the mass assigned to every choice $r_i$. The amount of mass that a student can assign to every choice is limited by the amount of mass left (he only has one unit of mass to distribute).

In Fig. 1(b), the student is almost sure that the answer to the question resides on $r_1$, leaves a small chance to $r_2$ and is almost sure that the last three options are incorrect. Still, he indicates that he is not completely confident on his answer ($u > 0$).

4 Conclusion

We propose ev-MCQs, an extension of classical MCQs that use belief function theory to allow students to express an imperfect answer. The acquisition of noise resulting from the constraint of giving a precise answer is prevented and a richer, closer image of the student’s current state of knowledge can be acquired.

We are currently organizing a set of experiments to validate the use of ev-MCQs in a CAT environment in order to diagnose student knowledge acquisition and misconception, by taking into account the different degrees of certainty, doubt and ignorance stated by the imprecise answers on ev-MCQs.

References